Teerawat Thewmorakot, ***Computability Theory on Polish Metric Spaces*****,** University of Connecticut, Storrs, CT, USA, 2023. Supervised by David Reed Solomon. MSC: 03D78. Keywords: computability theory, Polish metric spaces, index sets, classification problems.

Abstract

Computability theoretic aspects of Polish metric spaces are studied by adapting notions and methods of computable structure theory. In this dissertation, we mainly investigate index sets and classification problems for computably presentable Polish metric spaces. We find the complexity of a number of index sets, isomorphism problems, and embedding problems for computably presentable metric spaces. We also provide several computable structure theory results related to some classical Polish metric spaces such as the Urysohn space $\mathbb{U}$, the Cantor space $2^{\mathbb{N}}$, the Baire space $\mathbb{N}^{\mathbb{N}}$, and spaces of continuous functions.

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Waseet Kazmi, ***Ordered Groups, Computability and Cantor-Bendixson Rank*****,** University of Connecticut, Storrs, CT, USA, 2023. Supervised by David Reed Solomon. MSC: 03C57, 03D45, 06F15 Keywords: ordered groups, computability theory, computable groups, semidirect product, space of orders, Cantor–Bendixson rank.

Abstract

We study ordered groups in the context of both algebra and computability. Ordered groups are groups that admit a linear order that is compatible with the group operation. We explore some properties of ordered groups and discuss some related topics. We prove results about the semidirect product in relation to orderability and computability. In particular, we give a criteria for when a semidirect product of orderable groups is orderable and for when a semidirect product is computably categorical. We also give an example of a semidirect product that has the halting set coded into its multiplication structure but it is possible to construct a computable presentation of this semidirect product.

We examine a family of orderable groups that admit exactly countably many orders and show that their space of orders has arbitrary finite Cantor–Bendixson rank. Furthermore, this family of groups is also shown to be computably categorical, which in particular will allow us to conclude that any computable presentation of the groups does not admit any noncomputable orders. Lastly, we construct an example of an orderable computable group with no computable Archimedean orders but at least one computable non-Archimedean order.

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Daniel Mourad. ***Computability Theory: Constructive Applications of the Lefthanded Local Lemma and Characterizations of Some Classes of Cohesive Powers***. Department of Mathematics, University of Connecticut, Storrs, CT, USA. 2023. Supervised by David Reed Solomon. MSC: 03Dxx, 03C20. Keywords: local lemma, cohesive powers, Weihrauch reducibility.

Abstract

The Lovász local lemma (LLL) is a technique from combinatorics for proving existential results. There are many different versions of the LLL. One of them, the lefthanded local lemma, is particularly well suited for applications to two player games. There are also constructive and computable versions of the LLL. The chief object of this thesis is to prove an effective version of the lefthanded local lemma and to apply it to effectivise constructions of non-repetitive sequences.

The second goal of this thesis is to categorize some classes of cohesive powers. We completely describe both the isomorphism types of cohesive powers of equivalence structures and injection structures, as well as clarify the relationship between these cohesive powers and their original structures. We also describe the finite condensation of cohesive powers of computable copies of the integers as a linear order by cohesive sets whose complement are computably enumerable.

Finally, we investigate the possibility of decomposing problems in the Weihrauch degrees into a product of first order part and second order part. We give a preliminary result in this direction.

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