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The Principal Principle and the contingent a priori

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Abstract
In Chapter 6 of *Objects of Credence*, Anna Mahtani argues that the opacity of credence raises difficulties for the Principal Principle and proposes a revised principle relating credence and chance that avoids it. In this comment on her book, I both defend Mahtani's proposed principle against a charge of triviality and argue that the opacity of belief does not threaten the role of chance in guiding credence.

Keywords: Credence; chance; indeterminism; Principal Principle

In Chapter 6 of *Objects of Credence*, Mahtani considers the implications of the opacity of credence claims for our understanding of the relationship between credence and chance. One important such implication, she argues, is that David Lewis' (1987) Principal Principle, the canonical expression of the thought that chance is epistemically authoritative for credence, is unsustainable. This argument will be the focus of my attention, along with Mahtani's proposed revised version of the Principal Principle, one that requires credence to defer, not to chance, but to chance conditioned on all contingent a priori truths.

Mahtani's argument begins with an important observation. While credence is hyperintensional, chance claims are (merely) intensional. Mahtani's example: the chance of a fatality this summer on Mount Everest due to avalanches is the same as the chance of a fatality on Chomolungma, though someone might have a different credence for fatalities on the two, because they fail to recognise that Chomolungma and Mount Everest are the same mountain. This, Mahtani argues, creates a problem for the Principal Principle: how can a match between credence and chance be required if credences can differ in (hyperintensional) contexts where chances cannot?

To show how a misalignment between chance and credence threatens the Principal Principle, Mahtani draws on an informal argument of Hawthorne and Lasonen-Aarnio (2009). Suppose that a fair coin will be tossed and that if it lands

heads Ann will win the prize but if it does not, Bob will. Let us call the person who will actually win the prize 'Lucky'. Since either Ann or Bob will win the prize, either Lucky is Ann or Lucky is Bob. If Lucky is Ann, then Lucky's chance of winning, at the time of the coin toss, is that of Ann's. If Lucky is Bob, then Lucky's chance of winning is that of Bob's. Either way the chance of winning is one half; hence by the Principal Principle you should believe to degree one half that Lucky will win. But it is certain that Lucky will win; indeed, that Lucky will win is knowable a priori. So you should believe to degree one that Lucky will win, contrary to what is prescribed by the Principal Principle.

Hawthorne and Lasonen-Aarnio's argument raises many interesting questions, but it is not at all clear to me that it presents a direct challenge to the Principal Principle. For the distinct conclusions they draw respectively about rational credence and about chance in fact apply to both. Consider first the argument to the effect that the chance of Lucky winning is one-half (hereafter the Half Argument). Exactly the same argument could be made to support the conclusion that one's *credence* in Lucky winning should be one-half. For suppose, as seems reasonable, that one's credence in Ann and Bob winning is the same (namely, one-half). But Lucky is either Ann or Bob; so irrespective of which is the case, one's credence in Lucky winning should be one-half. And this is exactly what follows from the Principal Principle if the *chance* of Lucky winning is one-half.

What then about the argument that it is certain that Lucky will win (hereafter the 'Certainty Argument')? As Mahtani says, "if you are in any doubt about this, just consider how you would bet on the claim" (p. 101); in particular, whether there are any odds on Lucky losing so favourable that you would or should accept a bet on it happening. If the answer is 'no', then it seems not only that you are certain of Lucky winning but that there must be no *chance* of Lucky losing, for if there was some non-zero chance then you should accept a bet at sufficiently favourable odds. So again credence and chance align in the manner required by the Principal Principle.

What the Hawthorne and Lasonen-Aarnio argument shows then is not that the Principal Principle forces an alignment between chance and credence where there should not be one, but that in cases involving stipulated names we have apparently compelling grounds for two different probability assignments, whether these are objective (chances) or subjective (credences). If we take the sentence 'Lucky will win' to be true in all (and only) worlds w in which the person named Lucky in the actual world wins the lottery in world w, then both its chance and credence should be one half. If we take it to be true in all worlds w in which the person named Lucky in w, wins the lottery in w, then both its chance and credence should be one. Irrespective of which interpretation we give it, the credence one has in the proposition determined by the interpretation should align with its chance of truth. The upshot is that the problem their argument raises has nothing to do with the hyper-intensionality of credence. It does not arise because, in virtue of their opacity, 'Lucky will win the lottery' and 'Ann will win the lottery' are accorded different credences despite expressing the same proposition. As we have seen, they are not. Rather it arises because the different possible ways of mapping language to a propositional framework entail different attributions of both chance and credence to sentences.

Let's turn to Mahtani's proposed revision of Lewis' Principal Principle. Mahtani and Lewis take the object of both credence and chance to be propositions, which in

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turn are modelled as sets of possible worlds. In order for these possible worlds to settle the truth of both 'ordinary' claims and claims about the chance, they have to have some structure.[1] To make it explicit, let S be a set of possible states of the world, T be a set of possible times and H be the set of all corresponding histories, where a history is just a sequence of states, one for each moment of time, i.e. a function from times to states. Let H be the set of all such sets of histories and Δ be the set of all probability functions on H. A possible world is then a pair (h, Ch), where $h \in$ H is any history and Ch an assignment of a probability function $Ch_t \in \Delta$ to each time $t \in T$. Intuitively Ch represents a history of chances (i.e. how the chances are at each moment of time in history h), a history against which chance claims are evaluated.

For simplicity let's restrict ourselves to temporally indexed claims expressed by sentences like 'The coin will land heads at time t1", as all such claims are either truth or false at a given history. This is so even if the truth or falsity of the claim is indeterminate at some moment of time in that history, a claim being determinate or *settled* as true (false) in some history h at some time t if the claim is true (false) in every other history h' that shares a past up to t with h. What is or is not settled at a time is crucial to how we are to understand chance. If a claim is not settled at some time, then its chance of truth can be neither zero nor one. On the view that chance is a database expert, the opposite is true as well: if a truth is settled at a time, then its chance of truth at that time is either zero or one. In other words, chance *qua* database expert, is conditioned on all that is settled as true.

The simplest formulation of the idea that you should defer to chance simply says that you should set your credence in any proposition to what chance says its probability is. Formally, let $Ch_t(A) = x$ be the proposition that the chance of A at time t is x. Then:

Credence From Chance: For any rational credence function Cr, time t and proposition A such that $Cr(Ch_t(A) = x) \neq 0$:

$$Cr(A|Ch_t(A) = x) = x$$

Simple though it is, Credence From Chance contains all that we will require for subsequent discussion. But to grasp Mahtani's proposal we will also need a statement of (the second of) David Lewis' (1987) formulations of the Principal Principle. To that end, let H_t be the proposition that describes the complete history of the world up to time t and let Th be a complete theory of chance, i.e. the proposition that describes how the chances at each time depend on the history of the world up to that time. Note that, for each time t, $H_t \cap Th$ is the set of possible worlds (h, Ch), such that the sequence of states in h up till t are as H_t says and the sequence of chances determined by Ch are those described by Th, so that, for each time t, the proposition $H_t \cap Th$ determines a probability function $Ch_t \in \Delta$. Then according to Lewis:

Principal Principle: Let Cr be a reasonable initial credence function. Then for any time t and proposition A:

[1]Here I largely follow Belnap *et al.* (2001).

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$$Cr(A|H_t \cap Th) = Ch_t(A)$$

Mahtani notes that there are already a number of objections to the Principal Principle in the literature and endorses the revisions encoded in the New Principal Principle of Lewis (1994) which are designed to meet them. Since these developments are tangential to Mahtani's proposed revisions, I will ignore them. This allows for presentation of a simplified version of her proposal that credence should defer to chance conditioned on all a priori truths:

Mahtani's Principle: Let Cr be a reasonable initial credence function and $\hat{X}$ be the conjunction of all a priori truths. Then for any time t and proposition A:

$$Cr(A|H_t \cap T) = Ch_t(A|\hat{X})$$

For completeness, let's also give a formulation of her principle aligned with Credence from Chance and which does not require reference to the combinations of historical facts and history-to-chance laws that (on Lewis' account) determine them.

Mahtani's Principle (reformulated): For any rational credence function Cr, time t and proposition A such that $Cr(Ch_t(A|\hat{X}) = x) \neq 0$:

$$Cr(A|Ch_t(A|\hat{X}) = x) = x$$

Mahtani's proposed revision of the Principal Principle is compelling. If a proposition is a true a priori (whether necessarily or contingently so) then we are rationally required to believe it true. But if this is so then rational credence is always conditioned on the set of a priori truths and any requirement to defer to the chances should not conflict with this fact. Mahtani's Principle ensures that it does not. As a corollary it is immune to Hawthorne and Lasonen-Aarnio's objection to the Principal Principle.

As an aside, one might wonder whether the chances too should not always be conditioned on the a priori. This would be the case if chance were a database expert, for then, at any point in time, chance would be fully apprised of all that is true a priori at that time. In particular, if it is stipulated at t that the winner of the lottery is called Lucky then this fact would be recorded in the t-chances. So if chance is a database expert, then $Ch_t(\cdot|\hat{X}) = Ch_t(\cdot)$ for any measure, Ch_t, of the t-chances and Mahtani's principle just follows from the Principal Principle. But if chance is not a database expert, then Mahtani's Principle seems a better guide to the relationship between chance and credence than the Principal Principle.

Nonetheless, Mahtani argues, her principle has an apparent problem: chance functions conditioned on the set of a priori truths will assign measure one or zero to every proposition and so give little guidance in forming credences. To see why this is so, note that the proposition *actually P* is, if true, necessarily so, and if false, necessarily so. So its chance is either 1 or 0 depending on whether or not P is true. But a chance function conditioned on the a priori truth *actually P iff P* assigns the same chance to P as it does to *actually P*, i.e. 1 or 0. So you can defer to a particular chance function iff you are certain of either the truth or the falsity of every proposition. This, to say the least, seems odd. Knowing that a particular coin is fair

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hardly seems sufficient for determining that it will land heads or tails. But it seems we must accept this consequence if we adopt Mahtani's Principle, for chance functions assign measure one to all necessary truths and credence functions assign measure one to all a priori truths and from these two sets of truths all other truths can be inferred.

Mahtani thinks that this implication, odd or otherwise, serves to show that the hyperintensionality of credence (as opposed to the 'mere' intensionality of chance) requires us to rethink the role of chance as a source of expertise. But the role of hyperintensionality is far from apparent here and, in fact, I don't think that Mahtani's argument that her principle is trivial can be correct. For it does not follow from the fact that *actually P* is either necessarily true or necessarily false that its chance should be either zero or one. In particular, if is not settled whether or not *P*, then it is not settled whether or not *P* is actually true or false and hence whether *actually P* is either necessarily true or not. On the contrary if the chance of *P* is *x*, then the chance that *P* is actually true is also *x*.

These latter claims are emphatically denied by Mahtani (Forthcoming: 109): "Whether *P* is currently determinate or not, the claim *actually P* is determinate: if *P* is true, then *actually P* is true in all metaphysically possible worlds, and so has a chance of 1; and similarly if *P* is false, then *actually P* has a chance of 0." But this is not a conclusion we need accept. If *actually P iff P* is necessary and *P* is indeterminate then either *actually P* must be indeterminate (as I claim) or, contrary to initial supposition, *P* itself must be determinate (as Mahtani concludes).

A full assessment of which of these is correct is beyond the scope of this comment so let me conclude by drawing out its implications for the soundness of the Half and Certainty arguments. On the face of it the Certainty Argument can't be faulted. Take the chances to be defined on a set that includes the propositions that Ann/Bob/Lucky win the prize and that the coin lands heads/tails. Consider the partition of possible histories {Heads, Tails} induced by the latter pair of propositions. Its first member entails that Ann wins the prize, that Lucky is Ann and that Lucky wins the prize, the second that Bob wins the prize, that Lucky is Bob and that Lucky wins the prize. In neither of the two possible cases does Lucky lose. But if it is not possible for Lucky to lose then the chance that Lucky wins the prize must be one.

So what is wrong with the argument that the chance of Lucky winning is one-half? It is this. Although either Ann or Bob will win the prize, before the coin has landed it is not settled which it is. Similarly, although Lucky is either Ann or Bob, it is not settled which is the case until the coin lands. So the name 'Lucky' does not, at the time at which the chances are evaluated, rigidly designate one or the other. Or rather, it cannot, because at the relevant time it is not settled what the actual future is. But if it is not settled at the time of the coin toss who will win, then it seems that the stipulation that Lucky be the person who will win is not sufficient, at that time, to determine the reference of 'Lucky'. Likewise, although prior to the coin landing either necessarily Lucky is Ann or necessarily Lucky is Bob, it is not settled which it is; hence one cannot infer that all reasonable chance functions assign measure zero or one to these propositions.

Mahtani anticipates an argument of this kind but says that she is persuaded that there is a truth of the matter at the time of tossing as to who will win and that this suffices for dubbing the winner 'Lucky'. But my point is not that there can be no

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truth of the matter at the time of coin's tossing, but there is no *settled* truth of the matter at that time. What difference does this make? After all, in every possible history Lucky is either Ann or Bob and the chance of Ann and Bob winning is the same (namely, one-half). So even if it is not settled which of the two is Lucky, surely Lucky must have a chance of one-half of winning?

No. It is true that Lucky is either Ann or Bob and true that both Ann and Bob have a chance of one-half of winning. But, conditional on Lucky being Ann, the chance of Ann winning is one, not a half. Likewise, conditional on Lucky being Bob, the chance of Bob winning is one. So the chance of Lucky winning is one, as the Certainty Argument establishes. Or at least this must be the case if chance is a database expert in the sense of being apprised of all settled truths. For if we argue by supposition that one or other of Ann and Bob wins, then we must update the chances in line with this supposition. And in so doing we undermine the Half Argument. So we have indirect support for the view that chance is a database expert: it not only vindicates Mahtani's principle but it provides a solution to the puzzle posed by Hawthorne and Lasonen-Aarnio.

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